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Foot landmarking for footwear customization

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As consumers are becoming increasingly selective of what they wear on their feet, manufacturers are experiencing problems developing and fitting the right footwear. Literature suggests that shoes with a shape similar to feet may be comfortable because they attempt to maintain the feet in a neutral posture. The objective of this paper is to develop a metric to quantify mismatches between feet and lasts and also to be able to generate the two-dimensional outline of the foot using the minimum number of landmarks. Fifty Hong Kong Chinese were participants in the experiment. In addition to subject weight, height, foot length and foot width, the left foot outlines were drawn and 18 landmarks were marked on each of the two-dimensional foot outlines. A step-wise procedure was used to reduce the chosen 18 landmarks to eight, such that the mean absolute negative error (an indicator of 'tightness') between the foot outline and the modelled curve was 1.3 mm. These eight landmarks seem to show an improvement over those proposed by other researchers, thus showing the importance of choosing the right landmarks for modelling the foot. The positive and negative absolute errors were on average 1.8 mm and 1.3 mm respectively. Moreover, the mean errors for the toe region and for the rest of the foot were 1.7 mm and 1.6 mm respectively. The results indicate that the foot outline, an important component for footwear functionality and fitting, may be modelled using eight critical landmarks.

1. Introduction

True custom footwear is expensive to produce because of the complexity and the constraints imposed by the footwear manufacturing process (Cavanagh 1980). Some of these include the design and fabrication of the last (that is, the mould that gives the shoe its shape), the moulds for making the outsoles, the dies for cutting the upper patterns and so on (Cheskin 1987). As a result, mass customized shoes are designed by matching existing footwear to the foot outline of a person (e.g. Fernand 2002). Many footwear manufacturers adopt this technique to customize footwear (e.g. Nike 2002).

Unlike any other consumer product, personalized footwear or the matching of footwear to feet is not easy if delivery of comfortable shoes is to be the ultimate goal, even though footwear related discomfort is predominantly caused

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by localized pressure induced by a shoe that has a design unsuitable for that particular shape of foot (Parker 1996). Similar to achieving functional clearances for the diameter between a shaft and a bearing, the correct clearances between foot and shoe are necessary to achieve comfort. These clearances would depend on the stretch properties of the upper material, the cushioning characteristics of the midsole material, the construction method and so on. The environment in which the foot interacts with the shoe is not very conducive to maintaining any required 'clearance', as it can be extreme with high temperatures and high humidity levels during use. In addition, the lack of proper support for the foot will result in relatively high deformations due to the large number of bones in the foot. All of these factors will result in an expansion (or increase in volume) of the foot over time. In essence, even though the footwear provider may have designed-in an acceptable clearance, that clearance may not be appropriate over time. One solution to this problem may be that the foot and the weight of the wearer ought to be supported at a fixed number of points in a similar way to any other structure (Falter and Bögle 1998). In this way, the foot is held in the right places for proper functioning and, at the same time, allows additional clearance in other areas to take account of any deformation that may take place over time. Thus, it is important to capture the correct points on the foot for support. For example, the metatarso-phalangeal joint (MPJ) is important in the design of the flex groove so that the shoe flexes in the right place for good foot movement (Rossi and Tennant 2000). Parker (1996) recommends that footwear fitters (generally, shop staff) should ensure that footwear have the right fit for length, joints, toes, forefoot area, instep, quarters (that is, the shoe upper, which covers the rear part of the foot) and heel. Even though it is easy to find a pair of shoes that one can wear, they may not fit right at all parts of the foot. In addition, Liu *et al.* (1999) argue that differences in findings between foot shape studies are primarily due to the lack of a proper anatomically defined coordinate system. They argue that a coordinate system based on a few foot landmarks would be ideal. Their study was intended to locate a series of foot landmarks so that lengths and angles on the foot could be obtained for the design of foot lasts. However, Liu *et al.* did not focus on the shape of the foot, which is critical for footwear fitting (Goonetilleke and Luximon 1999, Rossi and Tennant 2000).

Feet are somewhat irregular in shape and footwear is designed to have smooth contours (Adrian 1991, Anzelc 1994). So fitting footwear involves matching an irregular shape to a regular shaped object. Thus, it would be useful if the correct support points on the foot could be located, so that they could match with those in the footwear. In this way, the rest of the foot and shoe could have some additional clearance to provide for the necessary expansion of the foot. By 'holding' the foot at the proper points or by reducing the degrees of the freedom of the foot within the shoe, the functionality of the foot might also improve.

Studies such as those of Ho (1998) and Liu *et al.* (1999) have discussed methods to obtain foot point data accurately and reliably in an attempt to capture the correct information for footwear fitting. However, the suitability of these landmarks has not been discussed. One important objective of the present study was to determine the critical locations on the foot so that a regular (or 'smooth') outline capturing foot shape could be produced. It is hoped that the landmarks chosen can serve as 'anchors' for matching footwear and for designing style into footwear. For the most part, fit in the third (height) dimension can be adjusted through lacing and straps that may be present. In addition, if the foot outline can be generated accurately, it will be useful in the determination of a robust anatomical coordinate system as proposed by Liu *et al.* (1999). In the present study, Non-uniform rational B-splines (NURBS), one of the most flexible B-splines for surface representation (Cheng and Barsky 1991, Choi 1991) was used to model the two-dimensional foot outline. B-splines are efficient curves and have many useful properties such as spatial uniqueness, boundedness and continuity, local shape controllability, and invariance to affine transformations (Bartels *et al.* 1987). Due to the affine properties of B-splines, translation and rotation are easy and hence the model can be easily manipulated (Bartels *et al.* 1987).

In order for a two-dimensional foot model to be useful, the methods for quantifying its value should also be considered. Least square error (Bartels *et al.* 1987, Cheng and Barsky 1991) and volume error (Yavatkar 1993) can be used to quantify the difference between feet and footwear. These errors provide information on the overall accuracy of a model. For footwear fitting, however, it is important to have an overall quantification as well as part quantification since pinching or tightness, even in a small area, can result in a consumer being dissatisfied with the footwear fit.

Thus the objective of the study was to determine the optimal number of landmarks to generate the important characteristics of the two-dimensional foot outline and to provide a meaningful error measure that could be used to classify the fit between feet and footwear.

2. Methodology

2.1. *Participants*

The participants in the experiment were 50 staff and students from the Hong Kong University of Science and Technology and the same as those whose data are reported in Goonetilleke and Luximon (1999). The age of the participants was not recorded, but all participants were Hong Kong Chinese males over 18 years and below 39 years. None of the participants had any foot disease or foot abnormality.

2.2. *Procedure*

After each participant completed a voluntary consent form, their stature and weight were recorded. While the participants were seated with both feet on the ground, their left foot outline was drawn on paper using a scriber (as described in Mochimaru and Kouchi 1997). Eighteen landmarks, including the location of the metatarso-phalangeal joint and the toes, were marked on this outline. The outlines that were drawn were then scanned at 300 pixels per inch. From the scanned image, 1000 (X, Y) coordinates of the foot outlines were obtained after coordinate extraction, thinning, curve fitting and uniform sampling (Goonetilleke and Luximon 1999). The thinning, curve fitting and sampling operations were done using the Surfacer (1995) software. When using a scriber, the line thickness of the foot outline was between 0.5 mm and 1 mm. Thus 1000 sampled points seemed to be appropriate to represent the foot outline, since the separation between the points was then 0.6 mm. This assumption was tested and is reported later. Programs written using Visual C++ and ACIS, a three-dimensional modelling kernel developed by Spatial Technology Inc, were used for further analysis. NURBS curves (Cheng and Barsky 1991) were then generated using the landmarks marked on the foot.

2.2.1. *Location of landmarks*. Unlike regular computer-aided design (CAD) models, there are important landmarks that ought to be captured when developing footwear, examples include the metatarso-phalangeal joints. The starting point for our study was 18 landmarks (figure 1(*a*)) on the foot outline. Seventeen of these 18 were selected from a 64-point study conducted by Ho (1998) (table 1). In addition to these 17, landmark 8 on the medial side of the 1st toe was included. Landmarks 1 and 10 were the locations of the lateral and medial metatarso-phalangeal joints, respectively. Landmark 15 showed the position of the heel, and landmarks 14 and 16 (located at the side of the calcaneus) were used to capture the curvature of the heel. Landmarks 2 to 8 were selected to provide information on the toe 'curvature'. Similarly, landmarks 11 and 13 were targeted to provide the two-dimensional arch curvature. Landmark 12 corresponded to the location of maximum arch height on the foot outline. Landmarks 17 and 18, around the cuboid and metatarsals, were chosen to capture the curvature on the lateral side of the foot.

2.2.2. *Errors in curve interpolation*. NURBS curves of weight equal to 1 were generated using the landmarks that were marked on the foot. The starting point for the curves was landmark 1 (figure 1(*a*)) and the curves were generated in a clockwise direction. After generating the curves using the chosen landmarks, the *distance* (or error) between the modelled curve and the actual foot outline were computed at each sampled point on the foot outline. The necessary number of

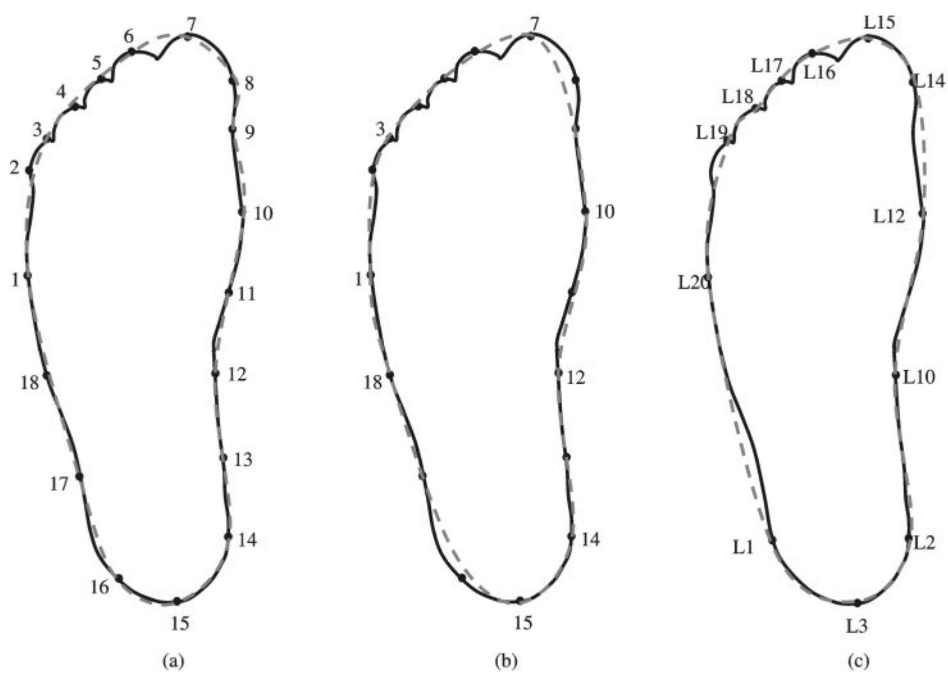

Figure 1. Landmark locations: (*a*) original 18 landmarks that included 17 landmarks from Ho (1998), (*b*) minimum set of eight landmarks, (*c*) Liu *et al.*'s (1999) 12 landmarks. 'L' is used to distinguish the Liu *et al.* landmarks from those used in the present study. The thick line shows an example of a foot outline and the dashed line is the modelled curve when using the corresponding set of landmarks.

Table 1. Mapping of landmarks from the present study to those in the Ho (1998) and Liu *et al.* (1999) studies. The numbers in parenthesis correspond to those used in the respective study. (See figure 1 for locations).

Landmarks	Description	Ho (1998)	Liu *et al.* (1999)
1	Point on lateral metatarso-phalangeal joint	(17)	(20)
2	Landmark on top of the inter-phalangeal joint (between distal and middle phalanx) of the 5th toe	(18)	
3	Tip of the 5th toe	(19)	(19)
4	Tip of the 4th toe	(20)	(18)
5	Tip of the 3rd toe	(21)	(17)
6	Tip of the 2nd toe	(22)	(16)
7	Tip of the 1st toe	(23)	(15)
8	Most prominent point at the side of the 1st toe	Not used	(14)
9	Landmark on top of the inter-phalangeal joint of the 1st toe	(4)	
10	Landmark on medial metatarso-phalangeal joint	(5)	(12)
11	Mid point between Ho's (1998) landmarks 5 and 7	(6)	
12	The highest point at the medial longitudinal arch	(7)	(10)
13	Mid point between Ho's (1998) landmarks 7 and 9	(8)	
14	Point on the medial side of the calcaneus	(9)	(2) 3 cm from the pternion
15	The outer most point on the calcaneus (medial view) or pternion	(11)	A point on the bisection line of the posterior calcaneus (3)
16	Point on the side of the calcaneus	(14)	3 cm from the pternion (1)
17	Point on the lateral side of the foot at 1/4 foot length	(15)	
18	Mid-point of 1 and 17 on the foot outline	(16)	

sampled points was determined using a procedure presented in section 3.2. There are many ways to compute the minimum distance between two polyhedra in two and three dimensions (Chin and Wang 1983, Dobkin and Kirkpatrick 1985, Edelsbrunner 1985). The shortest distance from each sampled point of the foot outline (C_1) to the modelled curve (C_2) (figure 2) (that is, *forward distance* $d(C_1, C_2)$ indicating that the distance is from C_1 to C_2) was defined to be the error, since it provided a conservative estimate (Surfacer 1995). Error could have also been calculated as the shortest distance from the modelled curve (C_2) to the foot outline (C_1) (that is, *backward distance* $d(C_2, C_1)$). In this case, some sections of the foot would not have been considered in the error calculation as shown in the partially unhatched region of figure 2(*b*).

The mean, maximum, and standard deviation of the errors were computed (the definitions and formulae are given in the appendix). If the modelled curve can be considered to represent the shape of the last (or shoe), then the overall mean error

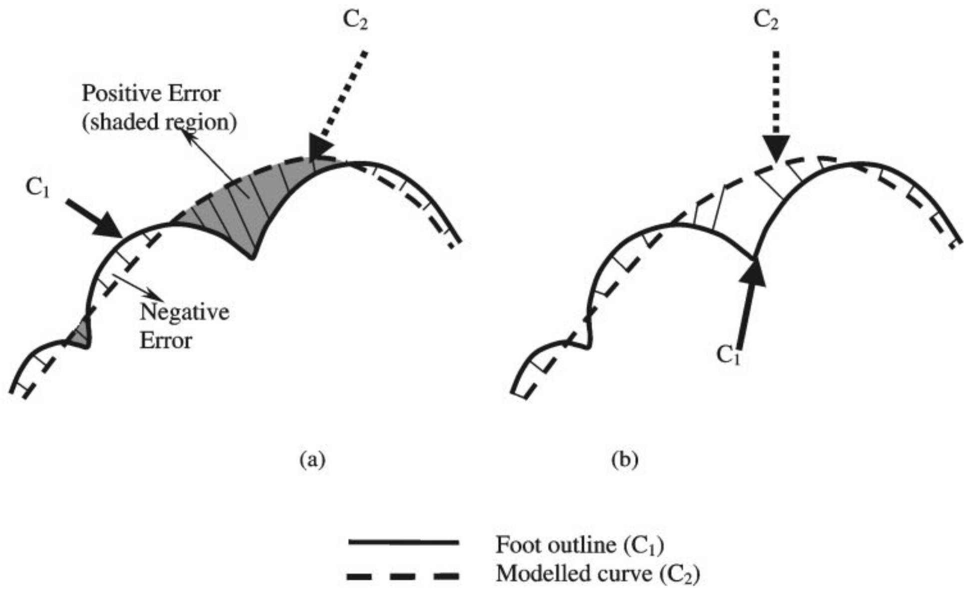

Figure 2. Illustration of errors from (*a*) foot outline to modelled curve, (*b*) modelled curve to foot outline. The unshaded portion in (*b*) between toe 1 and toe 2 can bias the magnitude of error. If the predicted curve is outside the foot outline, the difference (that is, the length of a line normal to the curve) is a positive error and a negative error is a difference when the predicted curve is inside the foot outline as shown. The shaded portions in (*a*) are all positive errors.

indicates 'overall fit'. The maximum error provides information on the largest level of 'misfit'. The standard deviation of the error provides information on the 'quality' of fit. The concept of *negative distance* has been used elsewhere to describe the extent of penetration in robot motion planning (Buckley and Leifer 1985, Cameron and Culley 1986). In a similar way, two types of error (positive and negative) as shown in figure 2 were used, based on the location of the modelled curve (see Appendix for the definitions and formulae). A positive error is one where the modelled curve is outside the foot outline, while a negative error is when the modelled curve is inside the foot outline (figure 2(*a*)). The differentiation of these two types of error is important for proper footwear fit. The positive error will result in a clearance or loose fit, while a negative error will result in interference or a tight fit. The proportion of the foot outline having positive error relative to that having negative error was also computed.

Apart from the negative and positive errors, the toe area and the rest of the foot outline were evaluated separately because shoes need more allowance at the toes than other parts (Parker 1996). The toe area was considered to be that from the tip of the fifth toe (landmark 3 in figure 1) to the tip of the first toe (landmark 7 in figure 1). One way to classify feet is by using the lengths of the first and second toes. Some people have the first (big) toe longer than the second (T12), while others have the second toe longer than the first toe (T21). Since this difference can influence footwear fit (Kreighbaum and Smith 1996), the results for the two groups of participants are presented separately.

3. Results

3.1. *Descriptive statistics*

The descriptive statistics of the participants are shown in table 2. The results indicate that approximately 18% of the Hong Kong Chinese population have their second toe longer than their big toe. For all participants, the mean weight, stature, foot length (heel-to-toe measurement of the foot along the Brannock measuring device) and foot width (side-to-side linear measurement across the ball of the foot as measured on the Brannock device) were 66.19 kg, 171.98 cm, 25.46 cm, and 9.32 cm, respectively. Foot length and foot width were measured from the foot outline.

3.2. *Determination of number of sample points for error calculation*

In order to determine the minimum number of sampled points needed to obtain a reasonable estimate for error, the number of sampled points was varied and the corresponding overall absolute error was computed (a plot with 18 points is shown in figure 3). A one-way ANOVA on the magnitude of error was performed for the 20-levels of sampled points when fitting a curve using the 18 landmarks. The levels of sampled points showed significant differences ($p < 0.05$) on the value of error. For the 18 landmarks, a *post hoc* Student-Newman-Keuls (SNK) test showed no significant difference when using 600 or more sampled points (as indicated in figure 4). In order to be conservative, 1000 points were used for all subsequent error computations.

3.3. *Determination of minimum number of landmarks*

The objective of the study was to predict the foot outline using the minimum number of landmarks. The process was started with a set of just two landmarks and a curve fitted to these two landmarks, and the error between the modelled curve and the actual outline was calculated. Thereafter, one landmark at a time was added to the chosen set of landmarks, and a new curve was fitted to that set of landmarks. The landmark to be retained depended on the landmark's contribution to minimize the absolute error between actual foot outline and the modelled curve. The starting set of two landmarks was chosen to be those on the metatarso-phalangeal joint (MPJ) (that is, landmarks 1 and 10), as these give an indication of where the foot bends, while giving additional information in relation to foot width. The error at each step was calculated and subjected to an ANOVA and a *post hoc* SNK test. For

Table 2. Descriptive statistics of participants in the two sub-groups T12 and T21.

	Weight (kg)	Stature (mm)	Foot length (mm)	Foot width (mm)
Toe 1 longer than toe 2 (T12) ($n=41$)				
Mean	66.8	1719.6	255.3	93.6
SD	11.82	63.64	12.44	5.20
Toe 2 longer than toe 1 (T21) ($n=9$)				
Mean	63.6	1720.7	251.2	91.4
SD	8.29	61.90	11.67	4.86

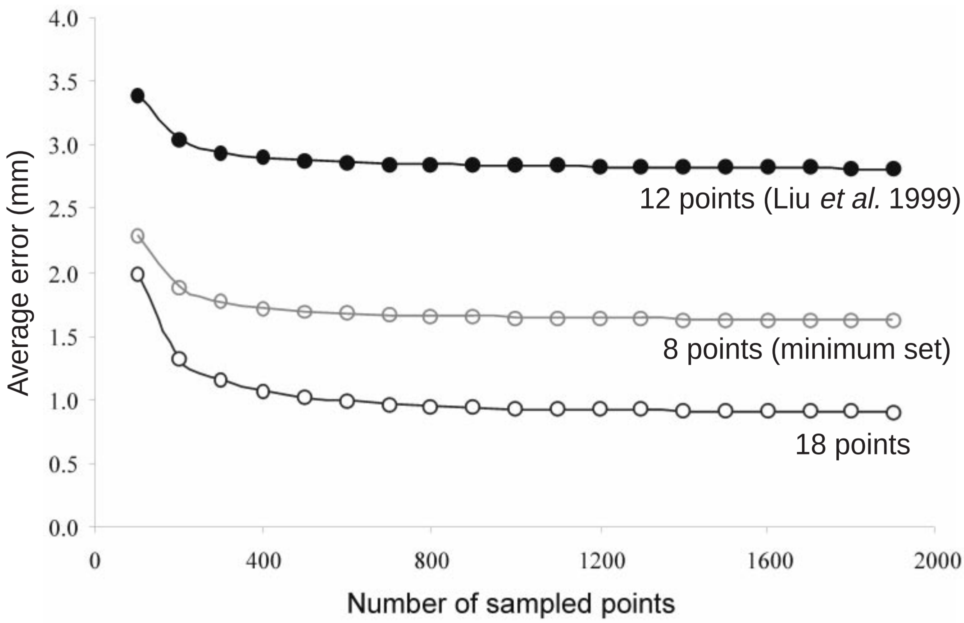

Figure 3. Variation of mean absolute error with changes in the number of sampled points when using the three different sets of landmarks.

	Number of points used for calculating error																			
	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
18 Landmarks	1.98	1.32	1.15	1.06	1.01	0.98	0.96	0.95	0.94	0.92	0.92	0.92	0.91	0.91	0.91	0.91	0.90	0.90	0.90	0.90
8 Landmarks (minimum. set)	2.29	1.88	1.77	1.72	1.69	1.67	1.66	1.65	1.65	1.64	1.64	1.63	1.63	1.63	1.63	1.62	1.62	1.62	1.62	1.62
12 landmarks (Liu et al.1999)	3.39	3.04	2.94	2.90	2.88	2.86	2.85	2.85	2.84	2.84	2.83	2.83	2.83	2.82	2.82	2.82	2.82	2.82	2.82	2.82

Note: Underlines indicate that the means are not significantly different at $p = 0.05$

Figure 4. Student-Newman-Keuls comparison of mean overall error for the 20 levels of sampled points using landmarks sets A, B, and C.

example, the error was calculated after fitting a curve using the landmarks {1, 10, and 2}, {1, 10, 3}, {1, 10, 4} . . . {1, 10, 18}. An ANOVA was performed using the magnitude of error and a *post hoc* SNK test was done to determine which combination had the lowest magnitude of error between the actual outline and the modelled outline. The results for the two groups of participants (that is those with toe 1 longer than toe 2 (T12) and those with toe 2 longer than toe 1 (T21)) are shown separately in figures 5 and 6, respectively. In iteration 1, set *a* (figure 5) represents the set of two landmarks, 1 and 10. Then, *a*,2 for example represents landmarks 1, 10 and 2. The top rows in both figures 5 and 6 have the highest magnitude of error for the indicated landmark set while those at the bottom have the lowest error. From figure 5, it can be seen that adding landmark 15 results in a significantly low overall error (iteration 1). For the participants whose second toe is longer than the big toe (T21), adding any one of landmarks 14, 15, or 16 results in no significant

Toe 1 > Toe 2

	Iteration 1			Iteration 2			Iteration 3			Iteration 4			Iteration 5			Iteration 6		
	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve
High	a,2	a,13	a,7	b,16	b,16	b,14	c,14	c,14	c,12	d,14	d,14	d,11	e,6	e,6	e,9	f,9	f,16	f,9
error	a,3	a,14	a,6	b,14	b,14	b	c,16	c,16	c,11	d,16	d,16	d,12	e,5	e,5	e,17	f,16	f,8	f,17
	a,9	a,15	a,5	b	b	b,16	c,13	c,13	c,17	d,11	d,17	d,8	e,4	e,4	e,4	f,6	f,6	f,4
	a,4	a,16	a,2	b,17	b,18	b,13	c,9	c,9	c,18	d,17	d,13	d,18	e,8	e,8	e,2	f,8	f,5	f,2
	a,8	a,12	a,3	b,18	b,17	b,17	c,17	c	c,16	d,9	d,9	d,9	e,9	e,13	e	f,5	f	f
	a,11	a,17	a,4	b,13	b,2	b,12	c	c,17	c,13	d,12	d,4	d,17	e	e,9	e,11	f,2	f,2	f,5
	a,5	a,8	a,9	b,12	b,11	b,18	c,12	c,8	c,9	d,13	d,5	d,2	e,11	e,16	e,13	f,4	f,11	f,11
	a,6	a,5	a,8	b,11	b,3	b,11	c,11	c,12	c,8	d,18	d,2	d,13	e,13	e	e,5	f	f,9	f,13
	a,7	a,6	a,11	b,2	b,13	b,2	c,8	c,11	c,14	d,2	d	d,6	e,16	e,11	e,8	f,17	f,4	f,8
	a,18	a,7	a,18	b,3	b,4	b,9	c,18	c,18	c	d,4	d,6	d	e,2	e,2	e,6	f,11	f,13	f,6
	a,12	a,4	a,12	b,9	b,12	b,3	c,6	c,6	c,2	d,5	d,18	d,5	e,17	e,17	e,18	f,13	f,17	f,16
	a,13	a,11	a,17	b,4	b,9	b,4	c,2	c,4	c,6	d,8	d,8	d,4	e,18	e,18	e,16			
	a,17	a,18	a,13	b,8	b,5	b,8	c,4	c,5	c,3	d,6	d,12	d,14						
	a,16	a,3	a,15	b,5	b,6	b,5	c,5	c,3	c,5	d	d,11	d,16						
Low	a,14	a,2	a,16	b,6	b,8	b,6	c,3	c,2	c,4									
error	a,15	a,9	a,14	b,7	b,7	b,7												

Figure 5. The Student-Newman-Keuls groupings for absolute error (overall, + ve and -ve) at each step (iteration) in the inclusion of landmarks for participants with toe 1 longer than toe 2 (T21). Iteration '0' or the starting point has 2 landmarks, 1 and 10. The rows are ordered such that the magnitude of error reduces from the top row to the bottom row. At each iteration, landmarks are 'added' to the previous set to minimize the magnitude of error. The stopping point (last iteration) can be based on the required magnitude of error. a = landmarks 1 and 10; b = landmarks 1,10, and 15; c = landmarks 1,10,15, and 7; d = landmarks 1,10,15,7,and 3; e = landmarks 1,10,15,7,3,12, and 14; f = landmarks 1,10,15,7,3,12,14,and 18. A vertical line indicates that the means are not significantly different at $p \leqslant 0.05$. + ve represents the mean positive error and -ve represents the mean negative error.

improvement in overall error as well as in the negative error (figure 6). In such cases, the inclusion of the landmark was determined by which landmark was common to both groups (T12 and T21) in order to keep a consistent pattern. Even though the addition of landmarks 14, 15, and 16 resulted in the same level of error for the T21 group (figure 6), the lowest significant error for the T12 group was with the addition of landmark 15 only (figure 5). Thus, iteration 2 was started using landmarks 1, 10, and 15 (called set b in figures 5 and 6). Similarly, the addition of landmark 7 in iteration 2 can result in a significant improvement in error for both groups. Hence landmarks 1, 10, 15, and 7 (set c) were chosen for iteration 3. For both groups, the addition of landmarks 3, 4, or 5 result in the same level of error with a $p < 0.05$ criterion. Landmark 3 was chosen since it captures the end curvature of the foot. Iteration 4 posed some problems. Here, adding landmarks 11 or 12 decreased the positive error significantly, but resulted in a significant increase in negative error, while adding landmarks 14 or 16 decreased negative error but resulted in a significant increase in positive error. Hence, two landmarks were included to form set e instead of just one: one to decrease positive error (landmark 12) and one to decrease negative error (landmark 14). Landmark 12 was selected because it captures the arch of the foot while landmark 14 on the same (medial) side was chosen to reduce its effect on the negative error. The set of landmarks for iteration 5 were hence 1, 10, 15, 7, 3, 12, and 14 (set e). The results showed that including

Toe 2 > Toe 1

	Iteration 1			Iteration 2			Iteration 3			Iteration 4			Iteration 5			Iteration 6		
	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve	Overall	+ve	-ve
High	a,2	a,13	a,6	b,16	b,16	b,14	c,14	c,14	c,16	d,14	d,14	d,11	e,4	e,5	e,2	f,16	f,6	f,2
error	a,9	a,14	a,7	b,14	b,14	b	c,16	c,16	c,17	d,16	d,16	d,12	e,5	e,4	e,9	f,2	f,16	f,9
	a,3	a,15	a,5	b	b	b,16	c,13	c,13	c,12	d,11	d,13	d,8	e,6	e,6	e	f,9	f,8	f,17
	a,8	a,16	a,2	b,17	b,18	b,13	c,9	c,9	c,11	d,12	d,17	d,18	e,9	e,9	e,11	f,6	f,5	f
	a,4	a,12	a,3	b,13	b,17	b,17	c,12	c	c,18	d,13	d,9	d,2	e	e,13	e,17	f	f,4	f,11
	a,11	a,17	a,4	b,18	b,11	b,12	c,17	c,12	c,13	d,9	d,6	d,9	e,11	e,8	e,13	f,17	f	f,4
	a,5	a,8	a,9	b,12	b,2	b,11	c	c,11	c,14	d,17	d,4	d	e,8	e	e,4	f,4	f,9	f,13
	a,6	a,5	a,8	b,11	b,13	b,18	c,11	c,8	c	d,8	d,5	d,13	e,13	e,11	e,18	f,11	f,2	f,16
	a,7	a,6	a,11	b,2	b,3	b,2	c,8	c,17	c,9	d,18	d,2	d,17	e,2	e,16	e,16	f,8	f,11	f,8
	a,18	a,4	a,18	b,9	b,12	b,9	c,18	c,6	c,8	d,2	d	d,6	e,16	e,2	e,8	f,5	f,17	f,5
	a,12	a,18	a,12	b,3	b,9	b,3	c,6	c,5	c,2	d,6	d,8	d,4	e,17	e,17	e,5	f,13	f,13	f,6
	a,13	a,3	a,17	b,4	b,4	b,8	c,2	c,4	c,3	d	d,18	d,5	e,18	e,18	e,6			
	a,17	a,11	a,13	b,8	b,5	b,4	c,3	c,18	c,4	d,4	d,12	d,14						
	a,16	a,2	a,15	b,5	b,8	b,5	c,5	c,3	c,5	d,5	d,11	d,16						
Low	a,14	a,7	a,16	b,6	b,6	b,7	c,4	c,2	c,6									
error	a,15	a,9	a,14	b,7	b,7	b,6												

Figure 6. The Student-Newman-Keuls groupings for absolute error (overall, + ve and -ve) at each step (iteration) in the inclusion of landmarks for participants with toe 2 longer than toe 1. Iteration '0' or starting point has two landmarks, 1 and 10. The rows are ordered such that the magnitude of error reduces from the top row to the bottom row. At each iteration, landmarks are 'added' to the previous set to minimize the magnitude of error. The stopping point (last iteration) can be based on the required magnitude of error. a = landmarks 1 and 10 and hence (a,2) implies landmarks 1, 10, and 2. Similarly b = landmarks 1,10, and 15; c = landmarks 1,10,15, and 7; d = landmarks 1,10,15,7, and 3; e = landmarks 1,10,15,7,3,12, and 14; f = landmarks 1,10,15,7,3,12,14, and 18. A vertical line indicates that the means are not significantly different at $p \leqslant 0.05$. + ve represents the mean positive error and -ve represents the mean negative error.

landmark 18 could significantly improve the error. Thereafter, the addition of other landmarks did not improve the level of error at a $p < 0.05$ significance level (see comparisons of set f with the other rows in iteration 6 of figures 5 and 6). Thus, the procedure was stopped with the sixth iteration.

The eight landmarks 1, 10, 15, 7, 3, 12, 14 and 18 can thus be called the *minimum set*, and this combination was then analysed further. The first analysis involved identifying the minimum number of sampling points needed for the error computations. A *post hoc* SNK test showed that there was no significant difference ($p < 0.05$) when 300 or more sampling points were used (see figure 4 and the 8-point curve in figure 3).

3.4. *Comparison of the minimum set with other sets*

The Liu *et al.* (1999) study used 26 landmarks on the leg, out of which 12 were on the foot outline. The minimum set of eight landmarks was then compared with the 12 proposed by Liu *et al.* (figure 1(*a*) and table 1). A SNK analysis of the 20-levels of sampled points showed that the use of 200 or more sampling points was sufficient for the error computations when using the landmarks proposed in Liu *et al.* (figure 4 and the 12-point curve in figure 3).

The three sets of the 18 original landmarks (set A),the eight landmark minimum set (set B) and the 12 landmarks proposed by Liu *et al.* (set C) were then compared.

Table 3. The mean absolute overall error (Mean), maximum absolute error (Max) and standard deviation (SD) of overall absolute error for the three different sets (A, B, and C) for the group T12 (toe 1 > toe 2). All errors in mm.

	18 landmarks – set A			Minimum set – set B			12 landmarks (Liu *et al.*) – set C		
Participant	Mean	Max	SD	Mean	Max	SD	Mean	Max	SD
1	0.95	8.49	1.06	1.43	7.94	1.24	2.93	15.91	3.84
2	0.90	5.91	0.88	1.49	6.74	1.37	2.99	15.80	3.91
3	0.87	8.83	1.10	1.30	6.70	1.03	2.90	14.44	3.81
4	0.83	4.37	0.68	1.96	7.07	1.64	3.34	15.24	4.02
5	0.72	4.88	0.59	1.56	5.69	1.43	2.24	11.67	2.80
6	0.90	5.16	0.92	1.53	5.89	1.41	2.76	11.79	3.00
7	1.09	7.17	1.06	1.78	6.98	1.40	3.43	16.15	3.96
8	0.80	5.39	0.81	1.47	6.72	1.44	2.36	11.72	2.81
9	0.87	5.51	0.85	1.75	8.35	1.83	2.90	12.57	3.08
10	1.02	8.29	1.15	1.65	8.44	1.65	2.35	10.08	2.37
11	0.95	6.06	0.85	1.59	6.16	1.33	2.77	13.08	2.98
12	0.99	7.67	1.04	1.30	6.35	1.03	3.25	15.37	4.17
13	0.76	5.02	0.72	1.94	8.44	2.11	2.11	11.13	2.67
14	1.08	9.10	1.30	1.95	9.59	1.65	2.86	13.74	3.27
15	0.83	5.58	0.71	1.17	6.35	1.13	1.87	8.74	2.06
16	0.85	5.76	0.88	1.54	6.97	1.40	2.53	11.20	2.78
17	1.01	8.08	1.12	1.91	8.26	1.79	2.88	12.24	3.02
18	0.90	6.61	0.98	1.84	8.33	1.77	3.33	14.35	3.56
19	0.93	5.65	0.77	1.58	5.64	1.29	3.19	16.07	4.02
20	0.98	8.33	1.06	1.75	8.39	1.65	2.87	12.64	3.12
21	1.03	5.15	0.94	1.71	5.91	1.36	2.97	13.16	3.15
22	0.81	7.13	0.83	1.49	6.27	1.12	2.53	11.08	2.66
23	0.93	5.31	0.78	1.81	8.22	1.52	3.31	14.81	3.83
24	0.85	7.25	0.85	1.35	6.32	1.15	2.62	13.08	3.17
25	1.04	5.67	0.89	1.72	8.07	1.26	3.10	13.81	3.35
26	0.86	5.46	0.76	1.63	6.96	1.42	3.04	12.48	3.19
27	0.81	5.16	0.78	1.32	5.25	1.15	2.69	13.23	3.36
28	0.79	4.72	0.73	1.63	5.96	1.48	3.02	14.75	3.60
29	1.02	8.18	1.17	1.79	6.09	1.54	3.03	13.68	3.30
30	0.95	7.88	1.00	1.71	11.21	1.77	2.71	11.58	2.93
31	0.93	6.69	0.90	1.20	8.05	1.02	2.48	12.81	3.14
32	0.80	5.07	0.70	1.46	6.39	1.43	2.68	11.77	3.05
33	0.84	4.27	0.80	1.46	6.77	1.31	2.74	13.22	3.37
34	0.92	7.26	0.97	2.00	8.36	1.85	3.05	13.32	3.48
35	0.91	4.88	0.82	1.82	5.71	1.40	2.36	11.24	2.70
36	1.10	6.88	1.14	2.30	7.86	1.92	3.14	12.78	3.10
37	0.80	7.11	0.84	1.60	6.31	1.39	3.21	16.39	4.06
38	0.77	4.63	0.66	1.43	6.13	1.25	2.27	9.95	2.59
39	0.80	5.22	0.68	1.65	5.47	1.25	2.31	12.13	2.86
40	0.86	8.06	1.07	1.41	5.53	1.18	2.84	14.33	3.64
41	0.95	5.63	0.80	1.07	5.39	0.86	2.94	14.72	3.75
Mean	0.90			1.61			2.80		
SD	0.10			0.25			0.37		

The overall error for each set is shown in table 3 (for group T12) and table 4 (for group T21). In the context of footwear fit, the mean overall error gives an indication of the overall fit. The maximum error can provide information on the area having the largest looseness or tightness. In addition, the standard deviation provides

Table 4. The mean absolute overall error (Mean), maximum absolute error (Max) and standard deviation (SD) of overall absolute error for the three different sets (A, B, and C) for the group T21 (toe 2 > toe 1). All errors in mm.

	18 landmarks – set A			Minimum set – set B			12 landmarks (Liu *et al.*) – set C		
Participant	Mean	Max	SD	Mean	Max	SD	Mean	Max	SD
1	0.93	6.53	0.90	1.56	6.06	1.31	2.61	12.11	2.87
2	0.97	8.67	1.15	1.79	5.89	1.47	2.36	11.36	2.58
3	1.28	8.18	1.57	2.21	7.50	1.55	3.02	11.45	2.70
4	0.96	9.86	1.20	1.44	8.18	1.28	2.96	12.10	3.34
5	1.23	11.08	1.37	1.84	6.58	1.49	3.39	14.63	3.63
6	0.85	7.23	0.92	1.63	8.20	1.59	3.12	14.32	3.68
7	1.04	8.38	1.10	1.98	8.35	1.86	3.53	14.54	3.86
8	1.14	9.28	1.23	1.64	11.17	1.75	3.17	14.84	3.83
9	0.82	3.52	0.67	1.88	6.95	1.57	2.83	12.29	3.10
Mean	1.03			1.78			3.00		
SD	0.16			0.24			0.37		

Table 5. ANOVA and *post hoc* Student-Newman-Keuls (SNK) comparison for mean overall absolute error (Mean), maximum absolute error (Max.) and standard deviation (SD) of absolute error for all participants ($n = 50$).

		SNK results		
	ANOVA results	18 landmarks (set A)	8 landmarks (minimum set) (set B)	12 landmarks (Liu *et al.*) (set C)
Mean	$F(2,147) = 729.89$, $p < 0.05$	0 .92	1.64	2.84
Max.	$F(2,147) = 252.11$, $p < 0.05$	6.64*	7.12*	13.12
SD	$F(2,147) = 665.82$, $p < 0.05$	0.94	1.44	3.26

*denotes that the means are not significantly different at $p < 0.05$.

important information on the variation from the mean. Hence, these three measures were used as dependent variables and all participants' data ($n = 50$) were then subjected to an ANOVA followed by a *post hoc* SNK comparison. The independent variable was the method with three levels (set A, set B, and set C). The ANOVA and SNK results are shown in table 5. The outline obtained using set C had a higher magnitude of error (mean = 2.84, maximum = 13.12, SD = 3.26, with $n = 50$) when compared to set B (minimum set) (mean = 1.64, maximum = 7.12, SD = 1.44, with $n = 50$) and set A (Mean = 0.92, maximum = 6.64, SD = 0.94, with $n = 50$).

As expected, set A was the closest to the actual outline (for T12, the mean error is 0.9 mm with standard deviation 0.1; for T21, the mean error is 1.03 with standard deviation 0.16) (tables 3 and 4) and was significantly different from both set B and set C. The curve with eight landmarks (set B) (T12, mean error = 1.61, SD = 0.25; T21, mean error = 1.78, SD = 0.24) outperformed that using 12 landmarks (set C) (T12, mean error = 2.80, SD = 0.37; T21, mean error = 3.00, SD = 0.37) showing that the locations rather than the actual number of landmarks play an important role when determining the foot outline.

3.5. *Positive and negative prediction errors*

Further analyses were performed on the minimum set (set B) and are shown in tables 6, 7, 8, and 9. Table 6 shows the absolute positive and absolute negative errors for

Table 6. The mean absolute overall error (Mean), maximum absolute error (Max), and standard deviation (SD) of absolute positive and absolute negative errors for feet with toe 1 longer than toe 2 (T12). All calculations were performed with the minimum set of eight landmarks. All errors in mm.

	Absolute positive error			Absolute negative error			Points contributing to positive error
Participant	Mean	Max	SD	Mean	Max	SD	(%)
1	1.77	7.94	1.39	1.07	4.78	0.92	52
2	1.88	6.74	1.63	1.09	2.97	0.86	51
3	1.46	6.70	1.10	0.91	2.42	0.70	70
4	2.09	7.07	1.82	1.74	4.22	1.26	63
5	1.80	5.69	1.58	1.10	4.09	0.94	66
6	1.94	5.89	1.60	1.01	3.32	0.88	57
7	1.88	5.58	1.27	1.64	6.98	1.55	56
8	1.80	6.72	1.57	0.85	3.78	0.83	66
9	2.10	8.35	2.03	1.13	5.27	1.19	65
10	1.97	8.44	1.85	0.98	2.76	0.74	68
11	1.88	6.16	1.46	1.21	4.26	1.02	56
12	1.38	6.35	1.06	1.17	3.91	0.98	60
13	2.55	8.44	2.39	0.90	2.87	0.78	63
14	2.32	9.59	1.75	1.12	3.91	0.99	69
15	1.42	6.35	1.27	0.70	2.68	0.61	65
16	1.69	6.97	1.52	1.13	2.88	0.87	73
17	2.02	8.26	1.96	1.57	3.95	1.06	76
18	2.01	8.33	1.99	1.58	5.75	1.35	60
19	1.39	5.20	1.02	1.91	5.64	1.60	63
20	1.98	8.39	1.78	1.00	2.53	0.77	76
21	1.75	5.91	1.41	1.63	4.10	1.25	65
22	1.69	6.27	1.16	1.09	3.30	0.93	66
23	1.96	8.22	1.57	1.53	5.30	1.40	66
24	1.54	6.32	1.21	0.87	2.98	0.81	73
25	1.77	8.07	1.33	1.64	4.11	1.14	60
26	1.88	6.96	1.52	0.98	3.21	0.82	73
27	1.61	5.25	1.29	0.84	2.80	0.65	63
28	2.09	5.96	1.68	1.15	4.98	1.05	51
29	2.13	6.09	1.64	1.06	4.07	0.98	69
30	1.92	11.21	2.01	1.33	3.99	1.13	65
31	1.42	8.05	1.29	1.01	2.69	0.62	47
32	1.84	6.39	1.53	0.58	2.04	0.49	70
33	1.69	6.77	1.48	1.13	3.51	0.93	59
34	1.76	5.96	1.55	2.32	8.36	2.16	58
35	2.13	5.71	1.51	1.12	2.80	0.72	70
36	2.72	6.78	1.88	1.73	7.86	1.84	58
37	1.75	6.31	1.44	1.28	5.55	1.23	68
38	1.69	6.13	1.40	0.97	2.70	0.70	64
39	1.86	5.47	1.30	1.24	4.23	1.02	67
40	1.44	5.53	1.20	1.39	4.15	1.14	56
41	1.17	5.39	0.92	0.99	3.29	0.80	43
Mean	1.83			1.21			63.0
SD	0.31			0.35			7.57

the modelled curve for group T12, while table 7 shows the errors for group T21. An ANOVA on the data for all participants showed that the absolute negative error (mean error = 1.32, SD = 0.43) was significantly smaller ($F(1,98) = 44.07$, $p < 0.001$) than the absolute positive error (mean error = 1.81, SD = 0.30). In general, for the 1000 sampling points, about 60% (T12 = 63.0 %; T21 = 58.7 %) of those points had a positive error. For the group having a longer first toe, the absolute positive error was 1.83 mm (SD = 0.31) and the absolute negative error was 1.21 mm (SD = 0.35) (table 6), while for group T21, the absolute positive error was 1.72 mm (SD = 0.22) and the absolute negative error was 1.82 mm (SD = 0.42) (table 7).

With a smooth curve, it is to be expected that the errors between the predicted curve and the actual foot outline will differ between the toe region and the rest of the foot due to the convexity of the toes and the convex intersections between toes. In order to understand the distribution of errors on the foot extremes, the foot was separated into a toe region (that is, first toe tip to fifth toe tip or from landmark 7 to 3) and the rest of the foot. Tables 8 and 9 show the error results for the two groups of participants when the foot outline was separated into two regions: toe region and the rest of the foot. According to the classification adopted, approximately 83% of the points on (T12 = 83.1%; T21 = 82.8%) the foot outline were considered as not belonging to the toe region (tables 8 and 9). An ANOVA for all participants ($N = 50$) showed no significant difference ($F(1,98) = 0.45$, $p = 0.5$) for overall error between the toe region (mean = 1.72, SD = 0.88) and the rest of the foot (mean = 1.63, SD = 0.30). For the group T12, the error at the toe region was 1.81 mm (SD = 0.91) and the error for the rest of the foot was 1.58 mm (SD = 0.29) (table 8), while for the group T21, the overall error for the toe region was 1.30 mm (SD = 0.56) and the overall error for the rest of the foot was 1.87 mm (SD = 0.27) (table 9).

An ANOVA of the absolute positive error for all participants showed no significant difference ($F(1,98) = 0.04$, $p = 0.94$) between the toe region (mean = 1.76, SD = 1.05) and the rest of the foot (mean = 1.76, SD = 1.38). For the group T12, the absolute positive error in the toe region was 1.89 mm (SD = 1.08) and 1.77 mm

Table 7. The mean absolute overall error (Mean), maximum absolute error (Max) and standard deviation (SD) of absolute positive and absolute negative errors for feet with toe 2 longer than toe 1 (T21). All calculations were performed with the minimum set of eight landmarks. All errors in mm.

	Positive error			Negative error			Points contributing to positive error (%)
Participant	Mean	Max	SD	Mean	Max	SD	
1	1.74	5.50	1.29	1.27	6.06	1.31	62
2	1.71	5.59	1.27	1.94	5.89	1.77	63
3	1.87	5.24	1.07	2.55	7.50	1.86	51
4	1.60	8.18	1.44	1.23	4.20	0.99	57
5	1.83	6.58	1.47	1.86	6.15	1.51	56
6	1.51	5.58	1.22	1.81	8.20	1.99	59
7	1.91	8.35	1.99	2.08	6.39	1.69	55
8	1.28	5.19	1.15	2.07	11.17	2.19	54
9	2.00	6.95	1.45	1.59	6.49	1.79	71
Mean	1.72			1.82			58.7
SD	0.22			0.42			6.12

(SD = 0.40) in the rest of the foot, while for the group T21 it was 1.19 mm (SD = 0.69) in the toe region and 1.78 mm (SD = 0.26) in the rest of the foot (tables 8 and 9). The absolute negative error in the toe region (mean = 1.03, SD = 0.66) of all participants was significantly smaller ($F(1,98) = 8.01$, $p = 0.0057$) than in the rest of the foot (mean = 1.35, SD = 0.47). For the group T12, the absolute negative error in the toe region was 0.99 mm (SD = 0.64) and in the rest of the foot was 1.22 mm (SD = 0.35) (table 8), while for the group T21, the absolute negative error at the toes was 1.17 mm (SD = 0.75) and in the rest of the foot was 1.96 mm (SD = 0.49) (table 9).

4. Discussion

The NURBS curve fitting technique (Cheng and Barsky 1991, Choi 1991) was used to model the foot outline. As expected, with a large number of points it was possible to accurately model irregular shapes. However, the objective was to be able to determine the outline within a given tolerance using as few landmarks as possible. Liu *et al.* (1999) in their study discussed methods to obtain foot landmark data accurately and reliably in an attempt to capture the right information for footwear fitting. One important aspect of footwear fitting is to be able to generate the foot outline on the 'footbed' plane. In the present approach, 18 landmarks were selected that could potentially describe the foot outline. An additive, step-wise procedure was used to minimize the difference between the actual curve and the modelled curve, starting with two of the important points for footwear fitting: the metatarso-phalangeal joint on the medial and lateral sides. The stepwise procedure resulted in eight landmarks to determine the foot outline. These included (figure 1(*b*)) the lateral MPJ (1), the medial MPJ (10), the tip of the first toe (7), the tip of the fifth toe (3), a landmark on the longitudinal arch (12), a landmark on the medial side of the calcaneus (14) to correct for arch curvature, the pternion (15), and a landmark on the lateral side around the midfoot (18). Generally, the flex line of a shoe is centred around the MPJ area (Rossi and Tennant 2000) so that the shoe has good flexibility. The big toe is generally critical in the determination of foot length, while the arch location can help in the provision of arch support. Shoe design, at present, places great emphasis on the shape of the heel contour (that is, the stiffener at the shoe's rear part) in an attempt to 'hold' the foot, as a stiff heel counter can prevent shape distortions. Indeed, the resulting landmarks have a known importance in shoe design and thereby in foot health.

The eight landmarks were then used to generate the foot outline with a mean overall error of approximately 1.6 mm. More importantly, the error measures calculated when using the selected eight landmarks were significantly lower than those with the 12 landmarks proposed by Liu *et al.* (1999). Thus, the importance of selecting the right landmarks for modelling irregular shaped objects such as the foot is highlighted. The classification into positive (that is, loose fit) and negative (that is, tight fit) errors allowed a greater understanding of the foot shape and the resulting 'looseness' or 'tightness' in the modelled curves. Loose or tight fit is important in the prevention of blisters and calluses (Cavanagh 1980, Cheskin 1987). The average absolute negative error (1.3 mm, $n = 50$) was significantly smaller than the positive error (1.8 mm, $n = 50$). Around 60% of the foot outline contributed towards the positive error, which can be attributed to the convex curve intersections between the toes. The negative error in the toe region was

Table 8. Mean overall, mean positive, and mean negative errors for the toe region and the rest of the foot for subjects with toe 1 longer than toe 2 (T12). All calculations were performed with the minimum set of eight landmarks. All errors in mm.

	Toe region				Rest of foot (rear)				
	Error			Points contributing to positive error	Error			Points contributing to positive error	Points contributing to rear part
Participant	Overall	+ve	−ve	(%)	Overall	+ve	−ve	(%)	(%)
1	1.37	1.74	1.08	44	1.44	1.78	1.07	53	85
2	1.02	0.27	1.12	12	1.57	1.93	1.07	57	87
3	1.61	1.63	0.08	99	1.24	1.42	0.92	65	85
4	3.27	4.18	1.43	67	1.78	1.78	1.78	62	88
5	0.84	0.74	0.94	48	1.67	1.91	1.14	69	87
6	2.82	3.17	1.15	83	1.36	1.67	1.00	53	88
7	2.64	1.77	2.96	27	1.63	1.89	1.22	61	86
8	0.85	0.83	0.85	26	1.58	1.86	0.85	72	86
9	1.65	1.82	0.68	85	1.81	2.32	1.20	54	66
10	1.04	1.09	0.97	53	1.77	2.09	0.98	71	84
11	1.62	2.61	1.13	33	1.58	1.81	1.24	60	86
12	1.25	1.35	1.10	59	1.30	1.38	1.18	61	86
13	1.43	1.12	1.60	36	2.01	2.66	0.69	67	87
14	1.47	1.88	0.96	56	2.05	2.39	1.17	72	84
15	1.65	1.70	0.35	96	1.10	1.35	0.71	60	87
16	2.05	2.68	1.10	60	1.45	1.56	1.14	75	86
17	1.84	1.82	1.85	54	1.92	2.04	1.47	79	86
18	2.26	2.61	0.64	82	1.59	1.39	1.77	47	63
19	2.57	1.46	3.04	30	1.45	1.39	1.57	67	88
20	3.21	3.50	0.59	90	1.49	1.65	1.03	74	85
21	1.55	0.82	1.99	38	1.73	1.83	1.51	70	86
22	1.67	1.74	1.39	80	1.37	1.65	1.00	57	61
23	1.28	0.28	1.58	23	1.89	2.04	1.51	72	87
24	2.04	2.18	0.77	90	1.25	1.41	0.88	70	86
25	1.84	2.23	0.82	73	1.67	1.54	1.84	55	72
26	4.26	4.47	0.13	95	1.26	1.38	1.00	69	88
27	2.35	2.49	1.14	90	1.17	1.41	0.83	59	87
28	0.75	1.08	0.48	45	1.76	2.23	1.27	52	86
29	1.77	2.02	0.72	81	1.80	2.14	1.09	67	87
30	3.90	4.00	0.42	97	1.34	1.35	1.34	59	86
31	0.65	0.83	0.56	33	1.30	1.49	1.11	50	85
32	0.40	0.47	0.33	49	1.62	1.98	0.65	73	87
33	2.42	2.80	0.77	81	1.31	1.43	1.16	56	86
34	3.48	3.74	0.35	92	1.77	1.22	2.37	52	87
35	1.86	2.06	1.03	81	1.80	2.17	1.15	64	64
36	1.47	2.03	0.58	61	2.45	2.85	1.91	57	85
37	0.96	1.00	0.24	95	1.69	1.91	1.30	64	87
38	0.90	0.60	1.03	29	1.50	1.75	0.96	69	88
39	1.32	1.45	0.74	82	1.86	2.22	1.37	57	62
40	2.40	2.59	1.57	81	1.26	1.16	1.37	52	86
41	0.52	0.59	0.48	31	1.15	1.23	1.09	45	87
Mean	1.81	1.89	0.99	62.6	1.58	1.77	1.22	62.2	83.1
SD	0.91	1.08	0.64	25.78	0.29	0.40	0.35	8.49	7.90

Table 9. Mean overall, mean positive, and mean negative errors for the toe region and the rest of the foot for subjects with toe 2 longer than toe 1 (T21). All calculations were performed with the minimum set of eight landmarks. All errors in mm.

	Toe region				Rest of foot (rear)				
	Error			Points contributing to positive error	Error			Points contributing to positive error	Points contributing to rear part
Participant	Overall	+ve	−ve	(%)	Overall	+ve	−ve	(%)	(%)
1	1.71	1.82	0.43	92	1.54	1.72	1.29	57	86
2	2.14	1.05	2.68	34	1.73	1.78	1.61	69	83
3	1.84	2.72	1.27	39	2.26	1.78	2.80	52	87
4	0.93	0.49	0.98	9	1.52	1.62	1.32	64	87
5	1.55	1.11	1.81	37	2.05	2.11	1.93	69	58
6	0.72	0.70	0.74	38	1.79	1.60	2.11	63	85
7	0.97	1.21	0.83	36	2.14	1.97	2.37	58	87
8	1.35	0.90	1.51	25	1.69	1.31	2.24	59	85
9	0.48	0.70	0.27	50	2.07	2.12	1.93	74	88
Mean	1.30	1.19	1.17	40.0	1.87	1.78	1.96	62.9	82.8
SD	0.56	0.69	0.75	22.47	0.27	0.26	0.49	6.93	9.45

1.03 mm ($n=50$) and was smaller than in the rest of the foot where the negative error was 1.35 mm ($n=50$). Further study is needed to understand better the distributions of the positive and negative errors and their use. The errors, however, can be used effectively to determine suitable materials and for 'padding' sensitive areas when designing shoes. Knowing the shape of the foot outline will allow designers to incorporate the fashion components more effectively and efficiently.

The maximum error for each modelled foot outline is relatively large (approximately 3.5 – 11.5 mm depending on the subject and the set of landmarks used; tables 3 and 4), which is primarily due to the somewhat irregular shape of the foot. The use of flexible materials and innovative design features in footwear can compensate for such high variations. The locations of the relatively higher errors can provide footwear designers with important insights for not only shape but also materials, overlapping areas, stitching and so on.

In summary, the foot outline can be modelled to any given level of error using selected landmarks but reductions in error are insignificant beyond a certain number of landmarks. Hence, the selected landmarks can be used as a lower bound so that designers and last makers can provide well fitting shoes. This study may be extended to refine the locations and possibly for generating the complete three-dimensional shape of the foot.

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References

Adrian, K. C. 1991, *American Last Making: Procedures, scale comparisons, sizing and grading information, basic shell layout, bottom equipment standards* (Arlington, MA: Shoe Trades Publishing).

Anzelc, D. 1994, *Practical Pattern Making* (Arlington, MA: Shoe Trades Publishing).

Bartels, R. H., Beatty, J. C. and Barsky, B. A. 1987, *An Introduction to Splines for Use in Computer Graphics and Geometric Modeling* (Los Altos, CA: Kaufmann Publishers).

Buckley, C. E. and Leifer, L. J. 1985, A proximity metric for continuum path planning, *Proceedings of the 9th International Joint Conference on Artificial Intelligence, Los Angeles, August 1985* (San Francisco: Morgan Kaufmann), 1096–1102.

Cameron, S. A. and Culley, R. K. 1986, Determining the minimum translation distance between two convex polyhedra, *Proceedings of the IEEE International Conference on Robotics and Automation, San Francisco, April 1986* (New York: IEEE), 591–596.

Cavanagh, P. R. 1980, *The Running Shoe Book* (Anderson World, CA: Mountain View).

Cheng, F. and Barsky, B. A. 1991, Interproximation: interpolation and approximation using cubic spline curves, *Computer-Aided Design*, **23**, 700–706.

Cheskin, M. P. 1987, *The Complete Handbook of Athletic Footwear* (New York: Fairchild Publications).

Chin, F. and Wang, C. A. 1983, Optimal algorithms for the intersection and the minimum distance problems between planar polygons, *IEEE Transactions on Computers*, **32**, 1203–1207.

Choi, B. K. 1991, *Surface Modeling for CAD/CAM: Advances in Industrial Engineering* (Amsterdam: Elsevier).

Dobkin, D. and Kirkpatrick, D. 1985, A linear algorithm for determining the separation of convex polyhedra, *Journal of Algorithms*, **6**, 381–392.

Edelsbrunner, H. 1985, Computing the extreme distances between two convex polygons, *Journal of Algorithms*, **6**, 213–224.

Falter, H. and Bögle, A. 1998, Conceptional Design of Renaissance Arch Bridges In A. Sinopoli (ed.), *Arch Bridges: History, analysis, assessment, maintenance and repair*. Proceedings of the Second International Arch Bridge Conference, Venice, Italy, 6–9 October 1998 (Rotterdam: A. A. Balkema), 45–52.

Fernand, 2002, http://www.fernandfootwear.com/TraceFeet.html. 21 June 2002.

Goonetilleke, R. S. and Luximon, A. 1999, Foot flare and foot axis, *Human Factors*, **41**, 596–607.

Ho, C.-F. 1998, 3-Dimensional foot digitization, Unpublished master's thesis, Hong Kong University of Science and Technology.

Kreighbaum, E. F. and Smith, M. A. 1996, *Sports and Fitness Equipment Design* (Champaign, IL: Human Kinetics).

Liu, W., Miller, J., Stefanyshyn, D. and Nigg, B. M. 1999, Accuracy and reliability of a technique for quantifying foot shape, dimensions and structural characteristics, *Ergonomics*, **42**, 346–358.

Mochimaru, M. and Kouchi, M. 1997, Automatic calculation of the medial axis of foot outline and its flexion angles, *Ergonomics*, **40**, 450–464.

Nike, 2002, http://nikeid.nike.com/. 21 June 2002.

Parker, K. 1996, *Settling Footwear Complaints* (Northants: SATRA Technology Centre).

Rossi, W. A. and Tennant, R. 2000, *Professional Shoe Fitting* (New York: Pedorthic Footwear Association with acknowledgement to the National Shoe Retailers Association).

Surfacer, 1995, *User's Guide – version 5.0* (Ann Arbor, MI: Imageware Inc.).

Yavatkar, A. S. 1993, Computer aided system approach to determine the shoe-last size and shape based on statistical approximated model of a human foot, Unpublished master's thesis, Tufts University, Medford, MA.

Appendix: Definitions and computation equations

Note that all references to points in this Appendix refer to the sampled points (used for calculating error) rather than the landmarks.
υ is the set of points (fx_i, fy_i) along the circumference of the foot outline (FO), where $i=1,..,N_f$. N_f is the total number of points on the FO.
ξ is the set of points (ax_i, ay_i) along the circumference of the modelled foot outline (MFO) where $i=1,..,N_a$. N_a is the total number of points on the MFO. The MFO is obtained by spline interpolation on selected points on the circumference of the FO (Choi 1991).
ζ is the set of points along the circumference of the FO that lie on or outside the boundary of the MFO.$\{(fx_i, fy_i) \mid \zeta \subset \upsilon$ and lie outside the MFO$\}$, where $i=1,..,N_n$. N_n is the total number of points in the set ζ.
ø is the set of points on the surface of the FO that lie inside the MFO$\{(fx_i, fy_i) \mid ø \subset \upsilon$ and lie inside the MFO$\}$, where $i=1,..,N_p\}$. N_n is the total number of points in the set ø.

It can be noted that the union of the sets ζ and ø give the set υ, $\{\xi \cap (ø=\upsilon\}$ *and* $\{\zeta \not\subset ø\}$.

The minimum distance from the point (fx_i, fy_i) on the FO to the MFO is given by $\delta_i = \min\{\sqrt{[\ (ax_j - fx_i)^2\ +\ (ay_j - fy_i)^2]}$, where $j=1,..,N_a\}$, for $i=1,..,N_f$.

he negative error at point (fx_i, fy_i) on the FO is given by $e_i = -\delta_i$ if $(fx_i, fy_i) \in ø$, while the positive error is given by $e_i = \delta_i$ if $(fx_i, fy_i) \in \zeta$, for $i\ =1,..,N_f$.
So the mean absolute error is given by $T = \Sigma|e_i|\ /N_f\ ,\ e_i \in \upsilon$, where $i=1,..,N_f$. The maximum error is given by $\max\{|e_i| \mid e_i \in \upsilon\}$, where $i=1,..,N_f$. The standard deviation of error is given by $\Sigma(e_i - T)^2\ /(N_f - 1)$, where $i=1,..,N_f$.
The mean absolute error of the positive (+ve) errors is given by $P = \Sigma|e_i|\ /N_p\ ,\ e_i \in ø$, where $i=1,..,N_p$. The maximum positive error is given by $\max\{e_i \mid e_i \in ø\}$, where $i=1,..,N_p$. The standard deviation of error is given by $\Sigma(e_i - P)^2\ /(N_p - 1)$, where $i=1,..,N_p$.
The mean absolute error of the negative (-ve) errors is given by $N = \Sigma|e_i|\ /N_{n,}\ e_i \in \zeta$, where $i=1,..,N_n$. The maximum negative error is given by $|\min\{e_i \mid e_i \in \zeta\ \}|$, where $i=1,..,N_n$. The standard deviation of error is given by $\Sigma(|e_i| - N)^2\ /(N_n - 1)$, where $i=1,..,N_n$.
The percentage of points contributing to positive error in tables 6 through 9 was calculated as $100^* N_p/(N_p + N_n)$ or $100^* N_p/N_f$.

v is the set of points on the surface of the FO that lie at the toe region {that is, $(fx_i, fy_i) \mid v \subset \upsilon$, where $i=1,..,N_t\}$. Let v_n be the set of points on the surface of the FO that are outside the MFO and lie at the toe region $\{(fx_i, fy_i) \mid v_n \subset \upsilon$ and $v_n \subset \zeta$, where $i=1,..,N_{tn}\}$. v_p is the set of points on the surface of the FO that are inside the MFO and lie at the toe region $\{(fx_i, fy_i) \mid v_p \subset \upsilon$ and $v_p \subset \varphi\}$, where $i=1,..,N_{tp.}$
The mean absolute error of the positive (+ve) errors at the toe region is given by $P_t = \Sigma|e_i|\ /N_{tp},\ e_i \in v_p$, where $i=1,..,N_{tp}$. The maximum positive error at the toe region is given by $\max\{e_i \mid e_i \in v_p\}$, where $i=1,..,N_{tp}$. The standard deviation of error is given by $\Sigma(e_i - P_t)^2/(N_{tp} - 1)$, where $i=1,..,N_{tp}$.

The mean absolute error of the negative (−ve) errors at the toe region is given by $N_t = \Sigma|e_i|/N_{tn},\ e_i \in v_n$, where $i = 1,..,N_{tn}$. The maximum negative error at the toe region is given by $|\min\{e_i|e_i \in v_n\}|$, where $i = 1,..,N_{tn}$. The standard deviation of error is given by $\Sigma(|e_i| - N_t)^2 / (N_{tn} - 1)$, where $i = 1,..,N_{tn}$.

σ is the set of points on the surface of the FO that lie at the rear region $\{(fx_i, fy_i) | \sigma \subset \upsilon$ where $i = 1,..,N_r\}$. Let σ_n be the set of points on the surface of the FO that are outside the MFO and lie at the rear region of the foot $\{(fx_i, fy_i)|\sigma_n \subset \sigma$ and $\sigma_n \subset \zeta\}$, where $i = 1,..,N_{rn}$. σ_{p} is the set of points on the surface of the FO that are inside the MFO and lie at the rear region of foot $\{(fx_i, fy_i)|\sigma_p \subset \sigma$ and $\sigma_p \subset \varphi\}$, where $i = 1,..,N_{rp}$. The mean absolute error of positive (+ve) errors at the rear region of the foot is given by $P_r = \Sigma|e_i|/N_{rp},\ e_i \in \sigma_p$, where $i = 1,..,N_{rp}$. The maximum positive error at the rear region is given by $\max\{e_i \mid e_i \in \sigma_p\}$, where $i = 1,..,N_{rp}$. The standard deviation of error is given by $\Sigma(e_i - P_r)^2/(N_{rp} - 1)$, where $i = 1,..,N_{rp}$.

The mean absolute error of the negative (−ve) errors at the rear region of the foot is given by $N_r = \Sigma|e_i|/N_{rn},\ e_i \in \sigma_n$, where $i = 1,..,N_{rn}$. The maximum negative error at the rear region is given by $|\min\{e_i \mid e_i \in \sigma_n\}|$, where $i = 1,..,N_{rn}$. The standard deviation of error is given by $\Sigma(|e_i| - N_r)^2/(N_{rn} - 1)$, where $i = 1,..,N_{rn}$.
The percentage of points contributing to the rear section, given in tables 8 and 9 was calculated as $100{*}N_r/(N_r + N_t)$ or $100{*}N_r/N_f$.